Math 433, Homework 10, Due November 5, 2006 at 2:00pm

From Curtis:
§26 6; §27 2, 3, 8, §28 1, 5, 10

Also:

1) Let

$$0 \xrightarrow{T_0} V_1 \xrightarrow{T_1} V_2 \xrightarrow{T_2} V_3 \xrightarrow{T_3} 0$$

be *exact*, that is the image of T_i equals the kernel of T_{i+1} for all i. Show the induced sequence

$$0 \xleftarrow{T_0^*} V_1^* \xleftarrow{T_1^*} V_2^* \xleftarrow{T_2^*} V_3^* \xleftarrow{T_3^*} 0$$

is *exact*.

2) Let V be a finite dimensional vector space over a field F, V^* its dual space, W a subspace of V and let $W^\perp$ be the annihilator of W inside V^*:

$$W^\perp = \{f \in V^* : f(w) = 0 \text{ for all } w \in W\}.$$

Define a linear transformation $\varphi : V^{**} \to (W^\perp)^*$ by $\varphi(T) = T|W^\perp$, i.e. we simply restrict the function $T : V^* \to F$ to the subspace $W^\perp$.

(a). Show that φ is surjective.

(b). Let $C : V \to V^{**}$ be the isomorphism defined in Homework 9, Problem 3. Show that $\mathrm{Ker}(\varphi \circ C) = W$ and write down an explicit isomorphism $V/W \cong (W^\perp)^*$.

3) Let V be a finite dimensional vector space over a field. What can you say about the function $L(V, V) \to L(V^*, V^*)$ given by $T \mapsto T^*$?

4) The field $\mathbb{C}$ can be viewed as a 2 dimensional vector space over the field $\mathbb{R}$. Let V be a finite dimensional vector space over $\mathbb{R}$. Show that $V \otimes_{\mathbb{R}} \mathbb{C}$ is a vector space over $\mathbb{C}$ by defining scalar multiplication of elements of $V \otimes_{\mathbb{R}} \mathbb{C}$ by elements of $\mathbb{C}$. What can you say about $\dim_{\mathbb{C}}(V \otimes_{\mathbb{R}} \mathbb{C})$?