

**MATH 332 - ALGEBRA AND NUMBER THEORY :  
HOMEWORK 3**

DUE FRIDAY, SEPTEMBER 14TH (NO LATER THAN **12PM**)

CHAPTER 2: CONGRUENCES

Solve the following exercises from Adler & Coury:

**Exercises (pages 64, 65):** 1, 4, 5, 7, 9, 12, 18, 22, 24, 27, 28.

Also:

- (1) Show that  $n^2 \equiv 0, 1$  or  $4$  modulo  $5$ , for all integers  $n$  (thus, you also need to show that there is no integer  $n$  such that  $n^2 \equiv 2$  or  $n^2 \equiv 3$  modulo  $5$ ).
- (2) Use part (1) to show that there are no integers  $x$  and  $y$  such that  $x^2 - 5y^2 = 2$ .
- (3) Show that if  $n \equiv 3 \pmod{4}$  then the equation  $x^2 + y^2 = n$  has no solutions with  $x, y \in \mathbb{Z}$  (hint: work modulo  $4$ ).

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