

MATH 103 - MATHEMATICAL EXPLORATIONS : HOMEWORK 4

DUE FRIDAY, FEBRUARY 23RD (IN CLASS)

2.6. The Irrational Side of Numbers.

Problem 1 (Square roots). First review the proof of the fact that $\sqrt{2}$ is irrational. Then write a complete proof of why $\sqrt{5}$ is irrational.

Problem 2 (Pi). Assuming the fact that the number $\pi = 3.1415\dots$ is irrational, prove that $\pi - 3$ is also irrational. (By the way, it took a *looonng* time to prove that π is irrational. This fact was first proved by Johann Lambert in 1761.)

Problem 3 (Irrational exponents). Let A be a number such that $3^A = 5$. Prove that A is irrational. (Hint: Suppose that A was rational, $A = \frac{n}{m}$, and so $3^{\frac{n}{m}} = 5$, but this last equation is impossible, why?).

Problem 4 (Squares). Suppose that α is a rational number. Is α^2 rational or irrational? Suppose that β^2 is an irrational number. Is β rational or irrational? Finally, suppose that δ is an irrational number. Is δ^2 necessarily irrational?

And a Challenge... The following problem is a challenge. **Do not** hand in a solution to this problem with your homework. If you solve it, you need to come by my office and tell me your solution. This will give you an extra 10 points in your final lecture + homework grade (out of 200). Here is the challenge:

Once upon a time I met two professors (Prof A and Prof M) who were extremely bright. Just to check how ridiculously clever they were I came up with a problem, for which they had to figure out the value of two natural numbers x and y , both bigger than 1 and $1 < x < y$. First, I called Prof A on the phone and only told him the sum of the two numbers (which, by the way, was less than 100, so $x + y < 100$) and then I called Prof. M and only told him the product of the two numbers. They were not allowed to tell each other what they knew, so the only information they had is what I told them. However, they called each other and this is how their conversation went:

Prof. M: Hello Prof. A, I do not know what numbers x and y are, even though Álvaro told me what their product is.

Prof. A: Hello Prof. M, Álvaro told me what their sum is, and I *was able to deduce* that you couldn't possibly know what the two numbers are. However, I do not know what x and y are.

Then Prof. M, full of joy, said: Thank you! Now I know what x and y are!

And Prof. A replied: Ha! Then I know them too!

Now, now, can YOU deduce the values of x and y ?