MATH 103 - MATHEMATICAL EXPLORATIONS : HOMEWORK 2

DUE FRIDAY, FEBRUARY 9TH (IN CLASS)

2.1. Counting (The Pigeonhole Principle).

Problem 1 (Only one cake)**.** Suppose we had a room filled with 370 people. Will there be at least two people who celebrate their birthday on the same day? Suppose there are 1100 people in the room. Are there 3 people who celebrate their birthday on the same day? Are there 4 people?

Problem 2 (Commuting)**.** One hundred people in your neighborhood always leave their home and start their commute to work between 7:30 and 8:00 a.m. and arrive to the office 30 minutes later. Why must two people always arrive at work at the same time, within a minute?

Problem 3 (RIP)**.** The Earth has 6.2 billion people and almost no one lives 100 years. Suppose this longevity fact remains true. How do you know that some year soon, more than 50 million people will die?

2.2. Numerical Patterns in Nature.

Problem 4 (Fifteen Fibonaccis)**.** List the first 15 Fibonacci numbers.

Problem 5 (Tons of ones)**.** Verify that $1 + \frac{1}{1+\frac{1}{1}} = 3/2$. Also, simplify $2 + \frac{2}{2+\frac{2}{2}}$.

Problem 6 (Baby bunnies)**.** This is the original problem suggested in the year 1202 by Leonardo of Pisa (a.k.a. Fibonacci). Suppose we have a pair of baby rabbits: one male and one female. Let us assume that rabbits cannot reproduce until they are one month old and that they have a one month gestation period. Once they start reproducing they produce a pair of bunnies each month (one of each sex). Assuming that no pair ever dies, how many pairs of rabbits will exist in a particular month? How many are there after 15 months? Fill out a chart like this:

Time in Months	Start (0)	1	2	3	4	5	6	...	15
Number of Pairs	1	1							

Here is a suggestion: draw a family tree to keep track of the offspring.

Problem 7 (Fibonacci relationships)**.** We'll use the symbol F_1 for the first Fibonacci number (so $F_1 = 1$), F_2 for the second Fibonacci number (so $F_2 = 1$), F_3 for the third Fibonacci number (so $F_3 = F_1 + F_2 = 1 + 1 = 2$), and so on. In other words, we write F_n for the nth Fibonacci number where n represents any natural number.

By experimenting with numerous examples in search of a pattern, determine a simple formula for $(F_{n+1})^2 + (F_n)^2$, that is, a formula for the sum of the squares of two consecutive Fibonacci numbers.

E-mail address: alozano@math.cornell.edu

Department of Mathematics (Malott 584), Cornell University.