Here are a few comments I had on HW 6. Note that these comments are not intended to serve as solutions. Comments on how the problem was scored are in purple italics.

**Chapter 9**

2. There is a short-cut: by Euler’s theorem it is not actually necessary to check whether \( a^4 \equiv 1 \mod 9 \) or whether \( a^5 \equiv 1 \mod 9 \) because \( \varphi(9) = 6 \) and 4 and 5 do not divide 6.

*I did not require that people know this for full points since Euler’s theorem comes after these exercises.*

8. One general strategy to show this is to show that \( k \) is common multiple of \( d \) and \( e \) if and only if \( a^k \equiv 1 \mod m \) then appeal to minimality.

29. Fermat’s Little Theorem is useful here. *Always be sure to cite the results you use.*

43. Use Euler’s Theorem (Theorem 6) and Proposition 7 noting that \( 322 = 20 \times 16 + 2 \).

*I only gave full credit here for computing the answer efficiently and otherwise a correct answer received a score of 2 out of 3.*

60. One strategy was to use induction to get the result for prime powers then show that if the result holds for \( n = a, b \) such that \( (a, b) = 1 \) then it also holds for the product.

Hint for a slick proof: Consider

\[
\frac{1}{n}, \frac{2}{n}, \ldots, \frac{n}{n}.
\]

Reduce the fractions and group them by (reduced) denominator \( d|n \).

*This one was not graded for rigor.*
Chapter 11

3. There are 5 such products.

*We can't assume abcd is well-defined in this question because that's what we're trying to show.*

11. Working in $U_{19}$,

\[
\langle 7 \rangle = \{ [1], [7], [11] \}
\]

\[
\langle 12 \rangle = \langle 8 \rangle = \{ [1], [8], [7], [-1], [-8], [-7] \}
\]

This answer is only unique up to choice of representatives mod 19.