Mathematics
Support
Capsules

Trigonometry
Basic Trigonometry

This module includes the following capsules:

0. Diagnostic Test
I. Angle Measurement
II. Trigonometric Ratios
III. Calculation of Easy Trig Ratios
IV. Graphs of Trig Functions
V. Obtaining & Using Trig Ratios for Other Angles.
VI. Basic Trig Identities
VII. Laws of Sines & Cosines
VIII. Addition Formulae
IX. Differentiating Sin θ, Cos θ
X. Inverse Trig Functions
XI. Post-Test
Answer the following questions without calculators or trig tables. (Leave answers like $53\pi$ or $\sin 13^\circ$ as is.)

**Questions**

1)

(a) $30^\circ =$

(b) $\frac{3\pi}{2}$ radians =

(c) $127^\circ =$

2)

(a) $\sin 60^\circ =$

(b) $\tan \left( -\frac{3\pi}{4} \right) =$

(c) $\sec \left( \frac{\pi}{2} \right) =$

3) Sketch the graph of $\sin x$.
(Make your vertical scale as large as possible.)

4) Given $\tan \theta = \frac{6}{7}$, find $\sin \theta$
5) Solve the following right triangle:
   (i.e., determine missing sides and angles.)

   [Diagram of a right triangle with sides labeled AB and AC]

   \[ \overline{AB} = \ldots \]
   \[ \overline{AC} = \ldots \]
   \[ \angle A = \ldots \]

6) Relate to \( \sin \theta \) and \( \cos \theta \)

   (a) \( \cos(-\theta) = \ldots \)
   (b) \( \sin\left(\frac{\pi}{2} - \theta\right) = \ldots \)
   (c) \( \sin 2\theta = \ldots \)

7) Express in terms of \( \sin \) and \( \cos \) of \( A \) and \( B \)

   \[ \sin(A - B) = \ldots \]

8) \[ \frac{d}{dx}(\cos 3x + \tan x) = \ldots \]

9) \[ \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \ldots \]

Check your answers on the next page!
# Trigonometry Diagnostic Answers

<table>
<thead>
<tr>
<th>Answers to trigonometry diagnostic test</th>
<th>If you missed these questions, review the indicated sections of the MSC Trigonometry Capsule.</th>
<th>If you have easy access to Keedy and Bittinger, <em>Algebra &amp; Trigonometry</em>, the relevant sections are:</th>
</tr>
</thead>
</table>
| 1) (a) $\pi/6$ radians  
(b) $270^\circ$  
(c) $127\pi/180$ radians | I. Angle Measurement | Chapter 8.1 |
| 2) (a) $\sqrt{3}/2 = 0.866\ldots$  
(b) 1  
(c) undefined | II. Trig Ratios  
III. Calculation of Easy Trig Ratios | Chapters 7.1, 7.2, 7.3, 7.4, 8.3 |
| ![Trigonometric Graph](image) | IV. Graphs of Trig Functions | Chapters 7.3, 7.4, 7.6 |
| 3) $-1$  
$0$  
$\frac{\pi}{2}$  
$\pi$  
$\frac{3\pi}{2}$  
$2\pi$ | | |
| 4) $6/\sqrt{85}$  
5) $\overrightarrow{AB} = (400/\cos 17^\circ)$  
$\overrightarrow{AC} = (400 \tan 17^\circ)$  
$\angle A = 73^\circ$ | V. Obtaining & Using Trig Ratios for Other Angles | Chapters 7.3, 8.5 |
| 6) (a) $\cos \theta$  
(b) $\cos \theta$  
(c) $2 \sin \theta \cos \theta$ | VI. Basic Identities | Chapters 7.5, 8.4 |
| 7) $\sin A \cos B - \cos A \sin B$ | VII. Laws: Sines & Cosines  
VIII. Addition Formulae | Chapters 7.7, 9.1, 9.2, 9.3 |
| 8) $-3 \sin 3x + \sec^2 x$ | IX. Differentiating Sin,Cos | See your calculus text! |
| 9) $\pi/6$ radians | X. Inverse Trig Functions | Chapters 9.4, 9.5 |

If you need more than the brief review of these capsules, we recommend Deborah Hughes-Hallett, *The Math Workshop: Elementary Functions* (W. W. Norton, 1980), Chapters 13–19. This is a detailed but very clear and conversational book, with excellent exercises.

These books are available for examination or browsing in the Mathematics Support Center and for sale at the Campus Bookstore and other bookstores in Collegetown.
The word "trigonometry", from ancient Greek, literally means "triangle measurement".

Angles are measured counterclockwise, from the x-axis, but different systems of units may be used.

Since the time of the ancient Babylonians, the angle of a full rotation has been divided into 360 degrees (and each degree into 60 minutes; each minute into 60 seconds).

However, in calculus a different system of units is used: radians.

\[
\text{radian measure} = \frac{\text{arc length}}{\text{radius}}
\]

Radian measure gives a pure number, since arc length and radius have the same units.

A full rotation in radian measure travels the circumference of a circle, so

\[
\frac{\text{arc length}}{\text{radius}} = \frac{2\pi r}{r} = 2\pi \text{ in full rotation.}
\]

Conversions: 360° is equivalent to 2π radians, so

\[
30^\circ \left(\frac{2\pi \text{ radians}}{360^\circ}\right) = \frac{\pi}{6} \text{ radians}
\]

and

\[
\frac{\pi}{6} \text{ radians} \left(\frac{360^\circ}{2\pi \text{ radians}}\right) = 30^\circ
\]

Choose \(\frac{2\pi}{360}\) or \(\frac{360}{2\pi}\) so that appropriate units cancel.
Exercises:

1. To change degrees to radians, multiply by ________.
2. To change radians to degrees, multiply by ________.
3. Convert the following degree measures to radians, and sketch the angle from the x-axis (e.g., □):

<table>
<thead>
<tr>
<th>degrees</th>
<th>90°</th>
<th>360°</th>
<th>45°</th>
<th>120°</th>
<th>-30°</th>
<th>540°</th>
<th>157°</th>
</tr>
</thead>
<tbody>
<tr>
<td>radians</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sketch</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. Convert the following radian measures to degrees, and sketch the angle from the x-axis:

<table>
<thead>
<tr>
<th>radians</th>
<th>π/4</th>
<th>-π</th>
<th>π/3</th>
<th>5π/6</th>
<th>7π</th>
<th>-5π/4</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>degrees</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sketch</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Answers: Check #1,2 by referring to other side of this sheet.

#3.

<table>
<thead>
<tr>
<th>degrees</th>
<th>90°</th>
<th>360°</th>
<th>45°</th>
<th>120°</th>
<th>-30°</th>
<th>540°</th>
<th>157°</th>
</tr>
</thead>
<tbody>
<tr>
<td>radians</td>
<td>π/2</td>
<td>2π</td>
<td>π/4</td>
<td>2π/3</td>
<td>-π/6</td>
<td>3π</td>
<td>3π/2</td>
</tr>
<tr>
<td>sketch</td>
<td>□</td>
<td></td>
<td>□</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#4.

<table>
<thead>
<tr>
<th>radians</th>
<th>π/4</th>
<th>-π</th>
<th>π/3</th>
<th>5π/8</th>
<th>π</th>
<th>-π/2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>degrees</td>
<td>45°</td>
<td>-180°</td>
<td>60°</td>
<td>150°</td>
<td>1260°</td>
<td>-225°</td>
<td>57°18'</td>
</tr>
<tr>
<td>sketch</td>
<td></td>
<td></td>
<td>□</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
There are two approaches to defining trigonometric ratios for a given angle $\theta$: using the lengths of the sides of a right triangle, or, using the coordinates of a point on a unit circle. These approaches are equivalent, but each has its advantages.

<table>
<thead>
<tr>
<th>Approach</th>
<th>Right Triangle</th>
<th>Unit Circle</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sine</strong>: $\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$</td>
<td>$y$-coordinate of $P$</td>
<td>$x$-coordinate of $P$</td>
</tr>
<tr>
<td><strong>Cosine</strong>: $\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$</td>
<td></td>
<td>$P(x,y)$</td>
</tr>
</tbody>
</table>

The circle handles angles greater than $90^\circ$ or $\pi/2$ radians; you can see symmetries and the proper signs (whether the ratio is positive or negative). You can calculate limits (in calculus).

Pythagorean theorem often helps. If you know two of the sides, you can find the third by

$$(\text{leg})^2 + (\text{leg})^2 = (\text{hypotenuse})^2$$

Show that the approaches are equivalent by the right triangle hidden in the circle diagram:

$$(\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{y}{1})$$

Sine and cosine are the most basic trigonometric ratios. There are, however, four others which can be defined in terms of sine and cosine:

- **Tangent**: $\tan \theta = \frac{\sin \theta}{\cos \theta}$
- **Secant**: $\sec \theta = \frac{1}{\cos \theta}$
- **Cotangent**: $\cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta}$
- **Cosecant**: $\csc \theta = \frac{1}{\sin \theta}$
Exercises

First, list the definitions: (and memorize any you don't know)

1. For a right triangle,
   \[ \sin \theta = \]
   \[ \cos \theta = \]
   sketch:

2. For a unit circle,
   \[ \sin \theta = \]
   \[ \cos \theta = \]
   sketch:

3. Furthermore,
   \[ \tan \theta = \]
   \[ \cot \theta = \]
   sec \theta =
   csc \theta =

Then,

4. For this right triangle,
   \[ \sin \theta = \]
   \[ \cos \theta = \]
   \[ \tan \theta = \]
   \[ \cot \theta = \]
   \[ \sec \theta = \]
   \[ \csc \theta = \]
   \[ \sin \phi = \]
   \[ \cos \phi = \]
   \[ \tan \phi = \]
   \[ \cot \phi = \]
   \[ \sec \phi = \]
   \[ \csc \phi = \]

5. Fill in the following table using the unit circle diagram:
   \[ \sin 0 = \]
   \[ \cos 0 = \]
   \[ \tan 0 = \]
   \[ \sin \frac{\pi}{2} = \]
   \[ \cos \frac{\pi}{2} = \]
   \[ \tan \frac{\pi}{2} = \]
   \[ \sin \pi = \]
   \[ \cos \pi = \]
   \[ \tan \pi = \]
   \[ \sin \frac{3\pi}{2} = \]
   \[ \cos \frac{3\pi}{2} = \]
   \[ \tan \frac{3\pi}{2} = \]
   \[ \sin 2\pi = \]
   \[ \cos 2\pi = \]
   \[ \tan 2\pi = \]

Answers: Check #1, 2, 3 by referring to other side of this sheet.

4. \[
\begin{align*}
\sin \theta &= \frac{3}{5} \\
\cos \theta &= \frac{4}{5} \\
\tan \theta &= \frac{3}{4} \\
\cot \theta &= \frac{4}{3} \\
\sec \theta &= \frac{5}{4} \\
\csc \theta &= \frac{5}{3}
\end{align*}
\]
All trigonometric functions are ratios, so the absolute lengths of the sides of a triangle do not matter. By the properties of similar triangles, these ratios are each constant for a given angle, regardless of the size of a triangle in which it may appear.

1. For several angles you can calculate the trigonometric ratios by sketching a triangle to show the sides in proportion, and using the Pythagorean theorem ($\text{hyp}^2 = \text{leg}^2 + \text{leg}^2$) to calculate the third side.

An isosceles right triangle has angles of $45^\circ$. If the legs are both 1, the hypotenuse must be $\sqrt{2}$.

An equilateral triangle can be split by an altitude into two $30^\circ$-$60^\circ$ right triangles. The hypotenuse is twice the short leg. Setting them equal to 2 and 1 respectively, ... 

Since all 3 sides are known for these triangles, all their trigonometric ratios may be calculated directly as shown in Trig. II.
2. For $0, \pi/2, \pi, 3\pi/2, \ldots$ the trigonometric ratios are easily calculated from the circle diagram, also shown in Trig. II.

3. These methods, coupled with symmetries that may be seen in the circle diagram, will cover calculation of all trigonometric ratios that are commonly thrown around without reference to tables.

e.g., \( \sin \frac{2\pi}{3} = \sin 120^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2} \)

\[
\cos \frac{2\pi}{3} = \cos 120^\circ = -\cos 60^\circ = -\frac{1}{2}
\]

Note that these ratios are sometimes negative - which ones can be seen in the circle diagram.
angle \( (\cos \theta, \sin \theta) \)

\[
\begin{align*}
(0, 1) & \quad \frac{\pi}{2} \\
(-\frac{1}{2}, \frac{\sqrt{3}}{2}) & \quad \frac{2}{3} \pi \\
(-\frac{\sqrt{3}}{2}, \frac{1}{2}) & \quad \frac{5}{6} \pi \\
(-1, 0) & \quad \pi \\
(-\frac{\sqrt{3}}{2}, -\frac{1}{2}) & \quad \frac{11}{6} \pi \\
(-\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}) & \quad \frac{7}{6} \pi \\
(0, -1) & \quad \frac{3}{2} \pi \\
(-\frac{1}{2}, -\frac{\sqrt{3}}{2}) & \quad \frac{4}{3} \pi \\
(-\frac{1}{2}, \frac{\sqrt{3}}{2}) & \quad \frac{2}{3} \pi \\
(1, 0) & \quad 0
\end{align*}
\]
Exercises: Review definitions, sketch relevant triangles, and fill in table with no reference to other trig tables or to calculations. This table may take some time to complete, but it is very worthwhile because these are numbers you'll meet again and again. You can shorten the time by looking for patterns of repetition.

<table>
<thead>
<tr>
<th>θ in degrees</th>
<th>in radians</th>
<th>sin θ</th>
<th>cos θ</th>
<th>tan θ</th>
<th>sec θ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>30°</td>
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<td></td>
</tr>
<tr>
<td>45°</td>
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<tr>
<td>60°</td>
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<tr>
<td>90°</td>
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<tr>
<td>120°</td>
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<tr>
<td>135°</td>
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<td></td>
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<tr>
<td>150°</td>
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<td></td>
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<tr>
<td>180°</td>
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<td></td>
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<tr>
<td>210°</td>
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<td></td>
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<tr>
<td>225°</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>240°</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>270°</td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>300°</td>
<td></td>
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<td></td>
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<tr>
<td>315°</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>330°</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>360°</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Use $\sqrt{2.1.4}$ and $\sqrt{2.1.7}$ to express answers to one decimal place. You can check your answers in RED.
Exercises: With the other side of this sheet, check your answers to the table you made in Trig. III, and use them to draw graphs from $0$ to $2\pi$ for each of the four functions listed. Note the period (interval of repetition) for each function.

\[ \sin \theta \]

\[ \cos \theta \]
Solutions to Exercises in TRIG III:

<table>
<thead>
<tr>
<th>θ in degrees</th>
<th>in radians</th>
<th>sin θ</th>
<th>cos θ</th>
<th>tan θ</th>
<th>sec θ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0 = 0</td>
<td>1 = 1.0</td>
<td>0 = 0</td>
<td>1 = 1.0</td>
</tr>
<tr>
<td>30°</td>
<td>π/6</td>
<td>1/2</td>
<td>√3/2</td>
<td>3 = √3</td>
<td>√3 = 1.73</td>
</tr>
<tr>
<td>45°</td>
<td>π/4</td>
<td>1/√2</td>
<td>1/√2</td>
<td>1 = 1.0</td>
<td>√2 = 1.41</td>
</tr>
<tr>
<td>60°</td>
<td>π/3</td>
<td>√3/2</td>
<td>1/2</td>
<td>√3 = 1.73</td>
<td>2/√3 = 1.15</td>
</tr>
<tr>
<td>90°</td>
<td>π/2</td>
<td>1 = 1.0</td>
<td>0 = 0</td>
<td>does not exist</td>
<td>does not exist</td>
</tr>
<tr>
<td>120°</td>
<td>2π/3</td>
<td>1/2</td>
<td>√3/2</td>
<td>-1/3 = -1.73</td>
<td>-2 = -2.0</td>
</tr>
<tr>
<td>135°</td>
<td>3π/4</td>
<td>1/2</td>
<td>-√3/2</td>
<td>-1 = 1.0</td>
<td>-√2 = -1.41</td>
</tr>
<tr>
<td>150°</td>
<td>5π/6</td>
<td>1/2</td>
<td>-√3/2</td>
<td>-1/3 = -1.73</td>
<td>-2/√3 = -1.15</td>
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<td>180°</td>
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<td>0 = 0</td>
<td>-1 = 1.0</td>
<td>0 = 0</td>
<td>-1 = 1.0</td>
</tr>
<tr>
<td>210°</td>
<td>7π/6</td>
<td>1/2</td>
<td>-√3/2</td>
<td>-1/3 = -1.73</td>
<td>-2/√3 = -1.15</td>
</tr>
<tr>
<td>225°</td>
<td>5π/4</td>
<td>1/2</td>
<td>-√3/2</td>
<td>-1 = 1.0</td>
<td>-√2 = -1.41</td>
</tr>
<tr>
<td>240°</td>
<td>4π/3</td>
<td>1/2</td>
<td>-√3/2</td>
<td>-1/3 = -1.73</td>
<td>-2/√3 = -1.15</td>
</tr>
<tr>
<td>270°</td>
<td>3π/2</td>
<td>0 = 0</td>
<td>does not exist</td>
<td>does not exist</td>
<td>does not exist</td>
</tr>
<tr>
<td>300°</td>
<td>5π/3</td>
<td>1/2</td>
<td>-√3/2</td>
<td>-1/3 = -1.73</td>
<td>-2/√3 = -1.15</td>
</tr>
<tr>
<td>315°</td>
<td>7π/4</td>
<td>1/2</td>
<td>-√3/2</td>
<td>-1 = 1.0</td>
<td>-√2 = -1.41</td>
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<td>330°</td>
<td>11π/6</td>
<td>1/2</td>
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<td>2π</td>
<td>0 = 0</td>
<td>1 = 1.0</td>
<td>0 = 0</td>
<td>1 = 1.0</td>
</tr>
</tbody>
</table>
Answers:

$\sin \theta$, period $= \pi$

$\tan \theta$, period $= \pi$

$\cos \theta$, period $= 2\pi$

$\sec \theta$, period $= 2\pi$

Note:
Dotted lines are asymptotes - functions are not defined for these points, so graphs never cross these vertical lines. (See GRAPHING capsules.)

Once you are familiar with these graphs, you should be able to recognize them or draw them on a moment's notice.

Note: Sec $\theta$ is reciprocal of Cos $\theta$.
You can see this in their graphs if you plot them on the same axes.
Where Cos $\theta = 0$, Sec $\theta$ has vert. asymptotes. Sec graph shoots vertically upward or downward near those values - you can see which by Cos graph to positive or negative at those points.
For first quadrant angles other than $0, \pi/6, \pi/4, \pi/3, \pi/2$ (or, $0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$) calculating the trigonometric ratios is a laborious process historically requiring very precise measurement and calculation. Fortunately this work has been done, and the results are listed in trigonometric tables, one of which appears with the next exercises. Many hand calculators are programmed (by infinite series) to give the basic trigonometric ratios.

Tables of trig ratios for $0^\circ$ to $90^\circ$ suffice to give trig ratios for any angle. By symmetries in the circle diagram, the functions for any angle can be related to those for a first quadrant angle.

Example: \[
\cos 166^\circ = -\cos(180^\circ - 166^\circ)
= -\cos 14^\circ
\]

Always remember the Pythagorean theorem (in a right triangle, $(\text{leg})^2 + (\text{leg})^2 = (\text{hypotenuse})^2$). It is very useful, & can sometimes save you running for a trig table.

Example: If $\cos \theta = \frac{5}{13}$, what is $\tan \theta$?

You can draw a right triangle including $\theta$, and use the Pythagorean theorem to find the missing side:

\[
\frac{\sqrt{13^2 - 5^2}}{5} = 12
\]

so $\tan \theta = \frac{12}{5}$. 

Sample problem, using trigonometry to solve a right triangle:

The navigator of an airplane flying over the ocean sights a small boat off his wing, at an angle of depression of 41°. From his altimeter, he reads that the plane is 3700 feet above the water. Will the plane receive a radio message if the boat's small transmitter has a range of only one mile?

First, draw a picture and label all information known and desired.

Note: angle B also measures 41°, for it is also an angle between the line of sight and a horizontal line.

From the picture we note the distance of the boat from the plane as c. We now have a right triangle with an angle of 41°, for which we either know or want to know the opposite side and the hypotenuse.

\[
\sin 41° = \frac{3700}{c} \quad \text{so} \quad c = \frac{3700}{\sin 41°} \approx \frac{3700}{.6560} = 5640. \text{ feet}
\]

Since a mile equals 5280 feet, the airplane will probably not hear the radio signal unless it moves toward the boat.
Exercises:

1. Without referring to a table, find the following:
   a. \( \tan \theta \) if \( \sin \theta = \frac{2}{5} \)
   b. \( \cos \theta \) if \( \cos \theta = 0.7 \)

For the following exercises, use the trig table below or your calculator, if it has trig functions.

2. A vertical flagpole 43 feet tall costs a shadow 61 feet long on ground. What is the angle of elevation of the sun, to the nearest degree?

3. A kite string makes an angle of 31° with the ground, which is level (i.e., not in Ithaca, probably), and 455 feet of string is out. How high is the kite?

4. A weather balloon is directly west of two observing stations 10 miles apart. The angles of elevation of the balloon from the two stations are 18° and 78°. How high is the balloon?

TRIGONOMETRIC TABLES

<table>
<thead>
<tr>
<th>Angle (Degrees)</th>
<th>Sin</th>
<th>Cos</th>
<th>Tan</th>
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</table>

Solutions are given on last page.
\[ \tan \theta = \frac{a}{b} \approx 0.4367 \]

\[ \cos \theta = \frac{7.14}{10} = 0.714 \]

\[ \tan \theta = \frac{43}{61} = 0.7049 \]

\[ \theta = 0.61 \text{ radians or } 35.18^\circ \]

\[ h = 0.515 \times 455 = 234 \text{ feet} \]

\[ \tan 18^\circ = \frac{h}{x+10} \quad \text{and} \quad \tan 78^\circ = \frac{h}{x} \]

\[ h = (x+10) \tan 18^\circ = x \tan 78^\circ \]

\[ x = \frac{10 \tan 18^\circ}{\tan 78^\circ - \tan 18^\circ} \]

\[ h = x \tan 78^\circ = \frac{10 \tan 18^\circ \tan 78^\circ}{\tan 78^\circ - \tan 18^\circ} \approx 3.5 \text{ miles} \]
The most important trigonometric identity of all time is

\[ \sin^2 \theta + \cos^2 \theta = 1 \]

by combination of the circle diagram and the Pythagorean theorem:

e.g., \((\sin 30^\circ)^2 + (\cos 30^\circ)^2 =\)
\[
\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 =
\]
\[
\frac{1}{4} + \frac{3}{4} = 1
\]

This identity often surfaces unexpectedly, so whenever you get hung up in any trig. problem, trot out \(\star\). You'll be surprised how often it's just what you needed.

From this \(\star\) identity, you can easily derive two others:

Exercise #1:

a) Divide \(\star\) identity by \(\cos^2 \theta\) throughout:

b) Divide \(\star\) identity by \(\sin^2 \theta\) throughout:

(If you will remember how to derive these from \(\star\), you'll never be confused trying to remember which term goes where in the results.)
There are many many other identities, most of which should not be memorized. Rather, you should get the idea and be able to figure them out for yourself.

Some identities, such as the reciprocal functions, come from the original definitions of the trig ratios. (See II).

**Exercise #2**

\[
\begin{align*}
\sin \theta &= \frac{1}{\csc \theta} \\
\csc \theta &= \frac{1}{\sin \theta} \\
\cos \theta &= \\
\sec \theta &= \\
\tan \theta &= \\
\cot \theta &= 
\end{align*}
\]

The cofunction identities arise from the fact that the right triangles formed in the circle diagram by complementary angles are congruent.

**Exercise #3**

\[
\begin{align*}
\sin \theta &= \cos \left( \frac{\pi}{2} - \theta \right) \\
\cos \theta &= \\
\tan \theta &= \\
\cot \theta &= \\
\sec \theta &= \\
\csc \theta &= 
\end{align*}
\]

Hint: after \( \sin \theta \) and \( \cos \theta \), resort to definitions in terms of \( \sin \) and \( \cos \).
Many more trig identities can be pulled from the circle diagram, relating to first quadrant angles:

Exercise #4 (Drawing little pictures can help. #1 is completed as an example.)

a) \( \sin \left( \theta + \frac{\pi}{2} \right) = \cos \theta \)

b) \( \cos (\theta + \pi) = \)

c) \( \tan (\theta - \pi) = \)

d) \( \sin (-\theta) = \)

e) \( \cos (-\theta) = \)

f) \( \tan (-\theta) = \)
Solutions to exercises:

Exercise #1

a) \( \frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta} \)

\[ \tan^2 \theta + 1 = \sec^2 \theta \]

b) \( \frac{\sin^2 \theta}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta} \)

\[ 1 + \cot^2 \theta = \csc^2 \theta \]

Exercise #2

\[ \sin \theta = \frac{1}{\csc \theta} \quad \csc \theta = \frac{1}{\sin \theta} \]

\[ \cos \theta = \frac{1}{\sec \theta} \quad \sec \theta = \frac{1}{\cos \theta} \]

\[ \tan \theta = \frac{1}{\cot \theta} \quad \cot \theta = \frac{1}{\tan \theta} \]

Exercise #3

\[ \sin \theta = \cos \left( \frac{\pi}{2} - \theta \right) \]
\[ \cos \theta = \sin \left( \frac{\pi}{2} - \theta \right) \]

\[ \tan \theta = \cot \left( \frac{\pi}{2} - \theta \right) \]
\[ \cot \theta = \tan \left( \frac{\pi}{2} - \theta \right) \]

\[ \sec \theta = \csc \left( \frac{\pi}{2} - \theta \right) \]
\[ \csc \theta = \sec \left( \frac{\pi}{2} - \theta \right) \]

Exercise #4

a) \( \sin \left( \theta + \frac{\pi}{2} \right) = \cos \theta \)

b) \( \cos \left( \theta + \pi \right) = -\cos \theta \)

c) \( \tan \left( \theta - \pi \right) = \tan \theta \)

d) \( \sin (-\theta) = -\sin \theta \)

e) \( \cos (-\theta) = \cos \theta \)

f) \( \tan (-\theta) = -\tan \theta \)
Law of Sines: \[ \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \]

Law of Cosines: \[ c^2 = a^2 + b^2 - 2ab\cos C \]

Note: if C is a right angle, the Law of Cosines reduces to the Pythagorean theorem.

These laws are listed here simply to make your basic trigonometry review complete.

For proofs, examples, and discussion, we recommend Deborah Hughes-Hallett, The Math Workshop: Elementary Functions (W.W. Norton, 1980), Ch. 15 on "the General Triangle", or any trigonometry text.

The Law of Cosines can be used on this triangle to find \[ \cos(A-B). \text{ Try it!} \]

(Answer given in VIII.)
\[ \sin(A+B) = \sin A \cos B + \cos A \sin B \]
\[ \cos(A+B) = \cos A \cos B - \sin A \sin B \]

If you memorize just these two formulae, you hold the key to a host of others that are intimately related by earlier identities (see VI.) such as \( \sin -\theta = -\sin \theta \); \( \sin \theta = \cos(\pi/2 \pm \theta) \); and so on. You will see these relationships as you do the exercises below.

A convenient starting place for a proof of all these identities is \( \cos(A-B) \), for which the formula can be derived using the Law of Cosines (See VII.) Then the relationships can be worked backward to provide proof for the addition formulae boxed above.

**Exercises:**

Using the addition formulae given above, complete the following identities:

1) \( \sin(A-B) = \sin(A+(-B)) = \)

2) \( \cos(A-B) = \)
3) \( \sin 2A = \)

4) \( \cos 2A = \)

5) Verify that \( \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \).

(Hint: put all \( \tan \)'s into \( \sin \) & \( \cos \).)

For the following two identities, start with \( \cos A = \cos 2\left( \frac{A}{2} \right) \) and verify that

6) \( \sin \frac{A}{2} = \sqrt{\frac{1 - \cos A}{2}} \)

(Hint: when you get hung up, remember ★ identity: \( \sin^2 \theta + \cos^2 \theta = 1 \))

7) \( \cos \frac{A}{2} = \sqrt{\frac{1 + \cos A}{2}} \)

Answers on next page
Answers to Exercises.

This can also serve as a Summary of Addition Formulae if you fill in the ones for which the formula was already given in the Exercises.

\[
\sin(A+B) = \\
\cos(A+B) = \\
1) \quad \sin(A-B) = \sin A \cos B - \cos A \sin B \\
2) \quad \cos(A-B) = \cos A \cos B + \sin A \sin B \\
3) \quad \sin 2A = 2 \sin A \cos A \\
4) \quad \cos 2A = \cos^2 A - \sin^2 A \\
5) \quad \tan(A+B) = \\
6) \quad \sin\left(\frac{A}{2}\right) = \\
7) \quad \cos\left(\frac{A}{2}\right) = \\
\]
For each of the following functions, graph the slope of the function on the extra blank graph below. (Eyeball where the slope is 0, +1, -1, positive, negative, etc., as shown):

\[
\text{Slope of } \sin \theta \quad (= \frac{d}{d\theta} \sin \theta)
\]

\[
\text{Slope of } \cos \theta \quad (= \frac{d}{d\theta} \cos \theta)
\]

(two points have been plotted from their slopes on the graph above)

Your resulting slope graphs should illustrate the facts, which are that \[\frac{d}{d\theta} \sin \theta = \cos \theta \quad \text{and} \quad \frac{d}{d\theta} \cos \theta = -\sin \theta.\]

These facts are proved in any calculus book; we show one version on the back of this sheet.

*Note: Calculus requires that \( \theta \) be measured in radians. (If degrees were used, the formulae would not be so simple.)
\[
\frac{d}{d\theta} \sin \theta = \lim_{\Delta \theta \to 0} \frac{\sin(\theta + \Delta \theta) - \sin \theta}{\Delta \theta} \quad \text{by definition of derivative.}
\]

\[
= \lim_{\Delta \theta \to 0} \frac{\text{adjacent}}{\text{hypotenuse}} \quad \text{in little gray "triangle" shown in diagram}
\]

\[
= \lim_{\Delta \theta \to 0} \cos \theta = \cos \theta
\]

For very small \(\Delta \theta\), we can treat this area as a triangle with hypotenuse \(\Delta \theta\) measured in radians.

So, \(\frac{d}{d\theta} \sin \theta = \cos \theta\) = \(\sin(\theta + \pi/2)\) (See Trig.VI-3.)

From this last equality,

\[
\frac{d}{d\theta} \cos \theta = \frac{d}{d\theta} \sin(\theta + \frac{\pi}{2}) = \sin((\theta + \frac{\pi}{2}) + \frac{\pi}{2}) = \sin(\theta + \pi) = -\sin \theta
\]

also shown on last line that \(\frac{d}{dx} \sin x = \sin(x + \frac{\pi}{2})\)

as in Trig VI-3

Thus, \(\frac{d}{d\theta} \cos \theta = -\sin \theta\)
Exercises: Find derivatives of the following functions by adding just the derivatives for \( \sin \) and \( \cos \), as boxed, to your repertoire of derivative rules: chain rule, product rule, quotient rule, etc.:

1. \[ \frac{d}{dx} \sin 2x = \]

2. \[ \frac{d}{dx} 2 \sin x = \]

3. \[ \frac{d}{dx} \sin^2 x = \]

4. \[ \frac{d}{dx} \sin^2 x = \]

5. \[ \frac{d}{dx} \tan x = \]

6. \[ \frac{d}{dx} \sec x = \]
Solutions:

1. $2 \cos 2x$ (chain rule, with $u = 2x$)

2. $2 \cos x$

3. $\frac{2 \sin x \cos x}{2u \ du/dx} = \sin 2x$ (chain rule, with $u = \sin x$)

4. $\frac{2x \cos x^2}{du/dx \cos u}$ (chain rule, with $u = x^2$)

5. $\frac{d}{dx} \frac{\sin x}{\cos x} = \frac{\cos x(\cos x) - \sin x(-\sin x)}{\cos^2 x}$

   $= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$

6. $\frac{d}{dx} \frac{1}{\cos x} = \frac{d}{dx} (\cos x)^{-1} = \frac{-\cos x}{u^2} \left(\frac{-\sin x}{du/dx}\right)$ (chain rule, with $u = \cos x$)

   $= -\frac{\sin x}{-\cos x} = \tan x \sec x$

General hint: since all trig functions can be expressed in terms of $\sin$ and $\cos$, you can always take derivatives without having to memorize any derivatives but $\sin$ and $\cos$. However, you must remember the minus sign in the derivative of $\cos$ — that little annoyance is essential.
Each trigonometric function has an inverse, defined as follows:

**Inverse Sine Function:**

\[ \text{arcsin} \ x = \text{the angle with sine equal to} \ x \]

\[ y = \arcsin x \text{ is equivalent to } x = \sin y \]

**Inverse Cosine Function:**

\[ \text{arccos} \ x = \text{the angle with cosine equal to} \ x \]

\[ y = \arccos x \text{ is equivalent to } x = \cos y \]

**Inverse Tangent Function:**

\[ \text{arctan} \ x = \text{the angle with tangent equal to} \ x \]

\[ y = \arctan x \text{ is equivalent to } x = \tan y \]

The thickened portion of each graph indicates the conventional principal values of \( y \) for each \( x \), which define an inverse trigonometric function. Thus \( y = \arcsin x \) and \( y = \arctan x \) have principal values \(-\pi/2 \leq y \leq \pi/2\), but \( y = \arccos x \) has principal values \( 0 \leq y \leq \pi \).

In calculus these values for \( y \) are always measured in radians, since radians are required whenever trig functions are differentiated or integrated.
Exercises: (Be sure to use principal values and radian measure.)

1. \( \sin^{-1} \frac{1}{2} = \)

2. \( \cos^{-1} \frac{1}{2} = \)

3. \( \tan^{-1} 1 = \)

4. \( \tan^{-1} -1 = \)

Answers:

1. \( \frac{\pi}{6} \)
2. \( \frac{2\pi}{3} \)
3. \( \frac{\pi}{4} \)
4. \( -\frac{\pi}{4} \)

(Principal values are shaded in circle.)
Answer the following questions without calculators or trig tables. (Leave answers like $53\pi$ or $\sin 13^\circ$ as is.)

Questions

1)

(a) $120^\circ =$
(b) $-3\pi$ radians =
(c) $68^\circ =$

1)

a. ________ radians
b. ________ degrees
c. ________ radians

2)

(a) $\cos 240^\circ =$
(b) $\tan \left(\frac{5\pi}{6}\right) =$
(c) $\csc(-3\pi/4) =$

2)

a. ______________
b. ______________
c. ______________

3) Sketch the graph of $\tan \theta$.
(Make your vertical scale as large as possible.)

3)

\[ \begin{array}{c|ccccccc}
0 & \frac{\pi}{2} & \pi & \frac{3\pi}{2} & 2\pi \\
\hline
\end{array} \]

4) Given $\sin x = \frac{4}{15}$, find $\sec x$.

4) ______________
5) Solve the following right triangles:
(i.e., determine missing sides and angles.)

\[ \triangle ABC \]

- \( BC = \) 
- \( AB = \) 
- \( BD = \) 
- \( AD = \) 
- \( \angle B = \) 

6) Relate to \( \sin \theta \) and \( \cos \theta \):

(a) \( \sin(-\theta) = \) 
(b) \( \sin\left(\frac{\pi}{2} + \theta\right) = \) 
(c) \( \cos 2\theta = \) 

7) Express in terms of \( \sin \) and \( \cos \) of \( A \) and \( B \):

\( \cos(-A + B) = \) 

8) \( \frac{d}{dx}(\cos^2 x - \cos x^2) = \) 

9) \( \sin^{-1}(-\sqrt{3}/2) = \) 

Check your answers on the next page!
<table>
<thead>
<tr>
<th>Answers to trigonometry post-test</th>
<th>If you missed these questions, review the indicated sections of the MSC Trigonometry Capsule.</th>
<th>If you have easy access to Keedy and Bittinger, <em>Algebra &amp; Trigonometry</em>, the relevant sections are:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) (a) 2π/3 radians</td>
<td></td>
<td>Chapter 8.1</td>
</tr>
<tr>
<td>(b) −540°</td>
<td>I. Angle Measurement</td>
<td></td>
</tr>
<tr>
<td>(c) 17π/45 radians</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) (a) −1/2</td>
<td>II. Trig Ratios</td>
<td>Chapters</td>
</tr>
<tr>
<td>(b) −1/√3</td>
<td>III. Calculation of Easy Trig Ratios</td>
<td>7.1, 7.2, 7.3, 7.4, 8.3</td>
</tr>
<tr>
<td>(c) −√2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3)</td>
<td>IV. Graphs of Trig Functions</td>
<td>Chapters 7.3, 7.4, 7.6</td>
</tr>
<tr>
<td>4) 15/√209</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5) ( \overline{BC} = 20' )</td>
<td>V. Obtaining &amp; Using Trig Ratios for Other Angles</td>
<td>Chapters 7.3, 8.5</td>
</tr>
<tr>
<td>( \overline{AB} = 40' )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \overline{BD} = 20\sqrt{2}' = 28.28' )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \overline{AD} = 20(\sqrt{3} - 1)' = 14.64' )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \angle DBC = 45° )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \angle ABD = 15° )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6) (a) − sin θ</td>
<td>VI. Basic Identities</td>
<td>Chapters 7.5, 8.4</td>
</tr>
<tr>
<td>(b) cos θ</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c) cos² θ − sin² θ</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7) ( \cos A \cos B + \sin A \sin B )</td>
<td>VII. Laws: Sines &amp; Cosines</td>
<td>Chapters 7.7, 9.1, 9.2, 9.3</td>
</tr>
<tr>
<td>8) ( -2 \cos x \sin x + 2x \sin x^2 )</td>
<td>IX. Differentiating Sin, Cos</td>
<td>See your calculus text!</td>
</tr>
<tr>
<td>( = -\sin 2x + 2x \sin x^2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9) −π/3 radians</td>
<td>X. Inverse Trig Functions</td>
<td>Chapters 9.4, 9.5</td>
</tr>
</tbody>
</table>
This picture is great! It summarizes practically all of the basic relations between the trigonometric functions at once.

Look closely. Since this is a unit circle (radius = 1), all the labels work by similar triangles and the following definitions:

- \( \sin \theta = \text{opp.}/\text{hyp.} \)
- \( \cos \theta = \text{adj.}/\text{hyp.} \)
- \( \tan \theta = \text{opp.}/\text{adj.} \)
- \( \cot \theta = 1/\tan \theta = \text{adj.}/\text{opp.} \)
- \( \csc \theta = 1/\sin \theta = \text{hyp.}/\text{opp.} \)
- \( \sec \theta = 1/\cos \theta = \text{hyp.}/\text{adj.} \)

- \( \tan \theta \) and \( \cot \theta \) even lie in a line tangent to the circle.

- Every right triangle in the picture gives you a trig identity by the Pythagorean theorem:
  
  \[
  \begin{align*}
  \sin^2 \theta + \cos^2 \theta &= 1 \\
  1 + \tan^2 \theta &= \sec^2 \theta \\
  1 + \cot^2 \theta &= \csc^2 \theta
  \end{align*}
  \]
  
  to the ridiculous (try this out on your friends in Engineering!)
  
  \[
  \csc^2 \theta + \sec^2 \theta = (\tan \theta + \cot \theta)^2
  \]

Points to ponder:

- How long is the arc from the x-axis to where the radius intersects the circle?
- How long would it be if the radius were 2?
- How would the labels in the picture have to change if the radius were 2? (The unit circle is nice, eh?)
- This picture shows \( \theta \) in the first quadrant. How would it be different if \( \theta \) were in each of the other quadrants?
- In particular, in each quadrant what happens to the sign (plus or minus) of:

<table>
<thead>
<tr>
<th>Quadrant</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin \theta )</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \csc \theta )</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \cos \theta )</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sec \theta )</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \tan \theta )</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \cot \theta )</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- If the x- and y- axes are labelled as usual, even most of the signs above "take care of themselves" from the picture. (i.e. The coordinate axes show you the sign directly.)

- Which do not?