INVERSE TRIGONOMETRIC FUNCTIONS.

1. FIND THE DERIVATIVE OF THE GIVEN FUNCTION W.R.T. THE INDEPENDENT VARIABLE.

(a) $y = \operatorname{Tan}^{-1} t^4$

(b) $z = t \operatorname{Cot}^{-1}(1+t^2)$

(c) $x = \operatorname{Sin}^{-1}\sqrt{1-t^4}$

(d) $s = \dfrac{t}{\sqrt{1-t^2}} + \operatorname{Cos}^{-1} t$

(e) $y = \operatorname{Sin}^{-1}\sqrt{x}$

(f) $z = \operatorname{Cot}^{-1}\dfrac{y}{1-y^2}$

3. FIND THE GIVEN INTEGRAL.

a) $\displaystyle\int \frac{dx}{\sqrt{25-4x^2}}$

(b) $\displaystyle\int \frac{dy}{36+4y^2}$

) $\displaystyle\int \frac{z\,dz}{5+2z^4}$

(d) $\displaystyle\int \frac{\sin x\,dx}{\sqrt{10-\cos^2 x}}$

) $\displaystyle\int \frac{dx}{\sqrt{5+4x-x^2}}$

(f) $\displaystyle\int \frac{7\,dx}{25-12x+4x^2}$

INVERSE TRIGONOMETRIC FUNCTIONS.

1. (a) $y = \tan^{-1} t^4$

$$\frac{dy}{dt} = \frac{d}{dt}\tan^{-1} t^4 = \frac{1}{1+(t^4)^2}\cdot\frac{d}{dt}t^4$$

$$\therefore \frac{dy}{dt} = \frac{4t^3}{1+t^8}.$$

(b) $z = t\cot^{-1}(1+t^2)$

$$\frac{dz}{dt} = \frac{d}{dt}\, t\cot^{-1}(1+t^2)$$

$$= \cot^{-1}(1+t^2) + t\cdot\frac{(-1)}{1+(1+t^2)^2}\cdot 2t$$

$$\therefore \frac{dz}{dt} = \cot^{-1}(1+t^2) - \frac{2t^2}{t^4+2t^2+2}$$

(c) $x = \sin^{-1}\sqrt{1-t^4}$

$$\frac{dx}{dt} = \frac{d}{dt}\sin^{-1}\sqrt{1-t^4}$$

$$= \frac{1}{\sqrt{1-(\sqrt{1-t^4})^2}}\cdot\frac{d}{dt}(\sqrt{1-t^4})$$

$$= \frac{1}{\sqrt{1-(1-t^4)}}\cdot\frac{1}{2}(1-t^4)^{-1/2}.(4t^3)$$

$$= \frac{1}{\sqrt{1-1+t^4}}\cdot\frac{1}{\sqrt{1-t^4}}\cdot -2t^3 = \frac{1}{t^2}\cdot\frac{1}{\sqrt{1-t^4}}\cdot -2t^3$$

$$\therefore \frac{dx}{dt} = \frac{-2t}{\sqrt{1-t^4}}$$

(1 d. - next page)

1(e) $y = \sin^{-1}\sqrt{x}$

$$\frac{dy}{dx} = \frac{d}{dx}\sin^{-1}\sqrt{x} = \frac{1}{\sqrt{1-(\sqrt{x})^2}}\cdot\frac{d}{dx}\sqrt{x}$$

$$= \frac{1}{\sqrt{1-x}}\cdot\frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x(1-x)}}.$$

$$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{x(1-x)}}$$

SOLUTIONS

1(d) $s = \frac{t}{\sqrt{1-t^2}} + \cos^{-1} t$

$$\frac{ds}{dt} = \frac{d}{dt}\frac{t}{\sqrt{1-t^2}} + \frac{d}{dt}\cos^{-1} t$$

$$= \frac{(\sqrt{1-t^2}).1 - t.\frac{1}{2}(1-t^2)^{-1/2}.-2t}{(\sqrt{1-t^2})^2} + \frac{(-1)}{\sqrt{1-t^2}}$$

$$= \frac{\sqrt{1-t^2} + t^2/\sqrt{1-t^2}}{(1-t^2)} - \frac{1}{\sqrt{1-t^2}}$$

$$= \frac{(\sqrt{1-t^2})(\sqrt{1-t^2})+t^2}{(\sqrt{1-t^2})(1-t^2)} - \frac{(1-t^2)}{(1-t^2)}\cdot\frac{1}{(\sqrt{1-t^2})}$$

$$= \frac{(1-t^2)+t^2-(1-t^2)}{(\sqrt{1-t^2})(1-t^2)} = \frac{t^2}{(1-t^2)^{3/2}}$$

1(f) $z = \cot^{-1}\frac{y}{1-y^2}$

$$\frac{dz}{dy} = \frac{d}{dy}\cot^{-1}\frac{y}{1-y^2}$$

$$= \frac{-1}{1+\left(\frac{y}{1-y^2}\right)^2}\cdot\frac{d}{dy}\left(\frac{y}{1-y^2}\right) = \frac{-1}{\frac{(1-y^2)^2+y^2}{(1-y^2)^2}}\cdot\frac{d}{dy}\left(\frac{y}{1-y^2}\right)$$

$$= \frac{-\cancel{(1-y^2)^2}}{(1-y^2)^2+y^2}\cdot\frac{(1-y^2).1-y.(-2y)}{\cancel{(1-y^2)^2}}$$

$$= \frac{-(1-y^2+2y^2)}{1-2y^2+y^4+y^2} = \frac{-(1+y^2)}{1-y^2+y^4}$$

$$\therefore \frac{dz}{dy} = \frac{-(1+y^2)}{1-y^2+y^4}.$$

1(a) $\int \frac{dx}{\sqrt{25-4x^2}}$

Use the formula $\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a} + C.$

$a = 5, \quad u = 2x.$

$\therefore \int \frac{dx}{\sqrt{25-4x^2}} = \sin^{-1}\frac{2x}{5} + C.$

(b) $\int \frac{dy}{36+4y^2}$

Use the formula $\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C.$

$a = 6, \quad u = 2y.$

$\therefore \int \frac{dy}{36+4y^2} = \frac{1}{6}\tan^{-1}\frac{2y}{6} + C = \frac{1}{6}\tan^{-1}\frac{y}{3} + C.$

(c) $\int \frac{z\,dz}{5+2z^4} = \int \frac{2z\,dz}{10+4z^4}$

$u = 2z^2, \quad du = 4z, \quad a = \sqrt{10}$

$\therefore \int \frac{z\,dz}{5+2z^4} = \frac{1}{2}\int \frac{2.2z\,dz}{10+4z^4} = \frac{1}{2\sqrt{10}}\tan^{-1}\frac{2z^2}{\sqrt{10}}$

2(d) $\int \frac{\sin x\,dx}{\sqrt{10-\cos^2 x}}$

Let $u = \cos x$, $\therefore du = -\sin x$, $a = \sqrt{10}$

$\therefore (-1)\int \frac{-\sin x\,dx}{\sqrt{10-\cos^2 x}} = -\sin^{-1}\frac{\cos x}{\sqrt{10}} + C$

(e) $\int \frac{dx}{\sqrt{5+4x-x^2}}$

Transform this into the form $\int \frac{du}{\sqrt{a^2-u^2}}$

$\therefore 5+4x-x^2 = 9-4+4x-x^2$
$= 9-(x^2-4x+4) = (3)^2-(x-2)^2$

$\therefore \int \frac{dx}{\sqrt{5+4x-x^2}} = \int \frac{dx}{\sqrt{(3)^2-(x-2)^2}} = \sin^{-1}\frac{x-2}{3} + C$

(f) $\int \frac{7\,dx}{25-12x+4x^2}$

Transform this into the form $\int \frac{du}{a^2+u^2}$

$25-12x+4x^2 = 16+9-12x+4x^2$
$= (4)^2+(2x-3)^2$ where $u = 2x-3$ and $du = 2$

$\therefore \int \frac{7}{25-12x+4x^2} = \frac{7}{2}\int \frac{2\,dx}{(4)^2+(2x-3)^2} = \frac{7}{8}\tan^{-1}\frac{2x-3}{4} + C.$