Algebra 1 Table of Contents

Chapter 1: Arithmetic to Algebra

1A: The Tables of Arithmetic
Students examine patterns in addition and multiplication tables to develop the rules for addition and multiplication, and extend the rules to negative integers. Students are challenged to work like mathematicians and to understand the rules they use.

1B: The Number Line
The basics of the number line are reviewed for integers and extended past the integers to include the rational and real numbers. The process of extension helps students use the number line to visualize addition and multiplication of any two numbers (not just integers), and to see that the basic rules they know carry over into these new sets of numbers.

1C: The Algorithms of Arithmetic
Students reexamine the algorithms they use to add, subtract, multiply, and divide in detail, to see how these algorithms use the basic rules of arithmetic, such as commutativity, associativity, and distributivity. Students are encouraged to look at the study of algebra as not only about finding a method that works but also understanding why that method works.

Chapter 2: Expressions and Equations

2A: Expressions
Students begin to transform algorithms, such as following simple number tricks, into expressions using variables. Variables are defined as placeholders that create a shorthand to express the patterns they find, and for understanding why those patterns occur.

2B: Equations
Equations are introduced to express relationships between expressions. Simple equations are solved using backtracking - a method for solving equations that has students think of the equation as a series of steps applied to a number $x$, and then undo each step in reverse order to find the initial value of $x$. This instinctive process will serve as a starting place for more formal equation solution.

2C: Solving Linear Equations
Students begin to formalize the basic method for solving equations. The Basic Rules and Moves of Equations are those operations that can be performed without changing the solution set of the equation, such as adding the same number to both sides of an equation and multiplying both sides of an equation by the same non-zero number. Students also explore why these moves don't change the solutions of the equation.

2D: Word Problems
Students learn the Guess-Check-Generalize method for building equations from situations. The method involves taking several guesses, checking those guesses against the text of the problem, and keeping careful track of the steps followed to check the answer. Finally, students guess with an arbitrary number (the variable) to build the equation. From there, they use the skills from earlier in the investigation to solve those equations.

Chapter 2 Extras

Chapter 3: Graphs

3A: Introduction to Coordinates
The coordinate plane is re-introduced as students experiment with transformations of points and shapes and explore absolute value and distance. Students also see how the two axes can represent different data points, and that graphs can show trends in data, a topic explored further throughout this chapter and in Chapter 4.

3B: Statistical Data
Students learn to use graphs, charts, and tables to summarize and interpret data. They recognize and construct visual representations of data, including box-and-whisker plots and scatter plots, and use their interpretations to make informed conclusions about data.

3C: Equations and Their Graphs
Students are shown that the graph of an equation is just another representation of the set of points that make the equation true. Simple and complex graphs are used to reinforce to students that no matter how difficult an equation is, it can be used as a point-tester to determine whether a particular point lies on the equation's graph.

3D: Basic Graphs and Translations
Students extend the concept of transformations from Investigation 3A, applying it to equations and their graphs. Simple linear translations of six types of equations are reviewed.
Transformations provide a link to Chapter 4, where the generic equation of a line will be

\[(y - k) = m(x - h)\] — a translation of the simpler equation \(y = mx\).

**Chapter 4: Lines**

4A: All About Slope
The study of slope begins by defining the slope between two points. They test for collinearity of sets of points by using the idea that three points are collinear if and only if the slope between each pair of them is the same. Ultimately, they prove the corollary that slope is invariant for all pairs of points on a line. They use this invariance as the point tester to see whether some given point is or is not on a line, and eventually (in the next investigation) to develop an equation for the line itself.

4B: Linear Equations and Graphs
The buildup from the previous investigation is resolved by having students use the point-tester concept to develop a general method for finding the equation of a line. This course does not emphasize any particular form of a linear equation, but rather works on the overriding principle that to graph a line, only two points on that line need to be found, and any two points will do.

4C: Intersections
Students learn to solve systems of linear equations using the substitution and elimination methods. While the explanation of these two methods is more or less traditional, the exposition relies on and emphasizes the basic moves and the point-tester concept. The proof that lines with the same slope are parallel introduces students to the concept of proof by contradiction.

4D: Applications of Lines
Students apply their work with lines to solve inequalities and estimate the line of best fit. Inequalities are explored by treating each side of the inequality as an equation to graph, and the inequality solution is found by comparing the \(y\)-heights of the two graphs. Fitting lines are found by determining the balance point of the data, and estimating the slope of the line. Students compare their lines with the actual data, calculating simple errors and thinking about how to minimize that error.

**Chapter 5: Functions - The Basics**

5A: Functions - The Basics
Functions are introduced as a machine defined by a specialized rule — one that assigns each input exactly one output. Students create their own rules for given sets of inputs and outputs, and from this foundation generate tables, algebraic expressions, and ultimately graphs. The lessons gradually add more formal algebra for expressing rules (such as \(f(x)\) notation and the concept of domains).

5B: Function and Situations
Students learn to fit functions to tables. First, they explore differences in successive outputs of a function, determining that constant differences imply linear functions. Next, recursive rules are introduced to describe some tables. Finally, these recursive rules are used to fit exponential functions to tables with constant ratios.

5C: Functions and Situations
Students extend their work from the end of Chapter 2 to build functions to model situations described in word problems using the Guess-Check-Generalize method.

**Chapter 6: Exponents and Radicals**

6A: Exponents
Following a similar process as in Chapter 1, students develop the basic rules of exponents, starting with positive integer exponents. The rules are used to find sensible definitions for zero and negative exponents.

6B: Radicals
Although most students are familiar with square roots prior to Algebra 1, the subject is treated more deeply here. Students learn the difference between rational numbers and irrational numbers and basic rules and conventions for calculating with square roots. The final lessons treat other radicals, such as cube roots, fourth roots, and more generally, \(n\)th roots.

6C: Exponential Expressions and Functions
Students explore exponential functions by looking at their graphs, exploring quotient tables (similar to the difference tables they saw in Chapter 5), calculating compound interest, and looking at graphs of exponential functions. These topics will be further investigated in Algebra 2.

**Chapter 7: Polynomials**

7A: The Need for Identities - Equivalent Expressions
The heart of this investigation lies in factors — forming them, expanding them, and ultimately using them in the development of the Zero Product Property. The exercises call upon all of the basic moves of equations students have learned and extend them to recognize equivalent expressions and develop algebraic identities.

7B: Polynomials and Their Arithmetic
Monomial and polynomial are introduced, and students explore the features of polynomial expressions. Extra practice is included for adding and multiplying polynomials, combining like terms, and factoring out the greatest common monomial factor of a polynomial.

7C: Factoring to Solve Quadratics
Algebra 2 Table of Contents

Chapter 1: Fitting Functions to Tables
1A: Tables
This investigation begins the Algebra 2 thread of functions and fitting. Students investigate how to use constant differences and other cues to fit linear and quadratic rules to input-output tables. They use a functional modeling language on their calculators to model and experiment with functions.

1B: Fitting the Data
Students begin to develop a statistical perspective, in the sense of thinking of data in terms of trends, rather than as individual points. Students investigate whether or not a data set can be reasonably approximated by a linear function, and they study alternatives to linear trends. Students investigate relationships among mean, median, variance and standard deviation.

1C: More About Recursive Models
Students encountered recursively-defined functions in Investigation 1A here, they investigate recursion in greater depth by analyzing a recursive function that determines the monthly payment on a loan. Students also investigate the factorial function — a recursively-defined function with no simple closed form.

Chapter 2: Functions and Polynomials
2A: About Functions
Students develop a "functional perspective:" what is a function? Students learn to deduce whether a given pairing is a function from a table, a graph, or an equation. Students also revisit notation and develop precise definitions of domain and range. Students investigate the arithmetic of functions, including composition. Inverse functions are introduced.

2B: Making it Fit
2C: Factors, Roots, and Zeros
The Factor Theorem and the Remainder Theorem are introduced and studied. Students learn the relationship between roots and factors of polynomials. Students also learn to divide polynomials by monic linear polynomials.

2D: Advanced Factoring
Students study various polynomial forms and methods for factoring them: chunking, scaling, and grouping. Rational expressions are also introduced.

Chapter 3: Complex Numbers
3A: Introduction to Complex Numbers
Students encounter problems whose solutions require calculations with square roots of negative numbers, and they then extend the real numbers to include these square roots. They investigate arithmetic of complex numbers.

3C: Complex Plane, Graphing, Complex Numbers
This investigation offers an exploration of some more advanced topics in complex numbers, including in-depth investigations of magnitude and direction, roots of polynomials, and roots of unity.

3B: The Complex Plane
Students learn how to graph complex numbers, and how to interpret arithmetic geometrically. Magnitude and direction are introduced.

Chapter 4: Linear Algebra
4A: Gaussian Elimination
Students solve systems of linear equations. Matrices and Gaussian elimination are introduced as tools for solving linear systems.

4B: Matrix Algebra
Students learn to solve matrix equations. Dot product is introduced. Students also learn to multiply matrices and find inverses of matrices, by hand and with their calculators.
4C: Applications of Matrix Multiplication
Students learn to use matrices to represent sequences of geometric transformations, model the evolution of a system over time, and analyze sequences of repeated probabilities.

Chapter 5: Exponential and Logarithmic Functions

5A: Working with Exponents
This investigation begins with a review of laws of exponents, including a review of zero and negative exponents. Students investigate arithmetic and geometric sequences, and they use these to extend the laws of exponents to include rational exponents.

5B: Exponential Functions
Students explore exponential functions via tables and graphs.

5C: Logarithmic Functions
Students learn what logarithms are, how to work with them, and how to graph logarithmic functions. Logarithmic scales are introduced.

Chapter 6: Graphs and Transformations

6A: Transforming Basic Graphs
Students enlarge their toolkit of basic graphs to include circles and graphs of simple cubic equations. They investigate how graphs are translated or stretched when the variables are changed with simple transformations.

6B: Affine Transformations
Students investigate the algebra of functions of the form \( x \rightarrow ax + b \). This algebra is applied to completing the square for quadratic equations and to reducing cubic equations to one of three simple forms.

6C: Graphing Using Affine Transformations
Students study an alternative way to understand the effect of translations and dilations on graphs—instead of transforming the graph they transform the axes.

Chapter 7: Sequences and Series

7A: The Need to Sum
Students investigate how to sum integers.

7B: Sum Identities
Students investigate definite and indefinite sums, and they develop the formula for the sum of the first \( n \) integers and sums of powers of a base.

7C: Arithmetic and Geometric Sequences and Series
Students see identities for sums of finite arithmetic and geometric series and for convergent infinite geometric series. The emphasis here is on the "linearity of summation."

7D: Pascal's Triangle and the Binomial Theorem
Students spend time investigating both Pascal's Triangle and the Binomial Theorem.

Chapter 8: Introduction to Trigonometry

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**Project: A Group of Functions**

**Chapter 7: Sequences and Series**

**Investigation 7A: The Need to Sum**

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**Investigation 7D: Pascal's Triangle and the Binomial Theorem**

Students spend time investigating both Pascal's Triangle and the Binomial Theorem.

**Project: The Line of Best Fit Contains the Centroid**

**Chapter 8: Introduction to Trigonometry**

**Lesson 8.0: Right Triangle Trigonometry—Optional Review**

**Investigation 8A: Trigonometric Functions**

Students learn the definitions of sine, cosine, and tangent as they apply to angles between 0 and 360 degrees, and learn to solve simple equations involving trigonometric functions.

**Investigation 8B: Graphs of Trigonometric Functions**

Students learn to graph sine, cosine and tangent. Some basic trigonometric identities are introduced, including the Pythagorean Identity.

**Investigation 8C: Applications to Triangles**

Students investigate the area of a triangle from a trigonometric perspective, and look at a geometric form of angle addition formulas. Law of Sines and Law of Cosines are introduced.