Conditional Probability and Markov Chains
Conditional Probability

- Conditional Probability contains a condition that may limit the sample space for an event.
- You can write a conditional probability using the notation
  \[ P(B|A) \]
  - This reads “the probability of event B, given event A”
Conditional Probability

The table shows the results of a class survey. Find $P(\text{own a pet} \mid \text{female})$

<table>
<thead>
<tr>
<th></th>
<th>yes</th>
<th>no</th>
</tr>
</thead>
<tbody>
<tr>
<td>female</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>male</td>
<td>5</td>
<td>7</td>
</tr>
</tbody>
</table>

14 females; 13 males

The condition female limits the sample space to 14 possible outcomes.

Of the 14 females, 8 own a pet.

Therefore, $P(\text{own a pet} \mid \text{female})$ equals $\frac{8}{14}$. 
Conditional Probability

The table shows the results of a class survey. Find $P(\text{wash the dishes} \mid \text{male})$

<table>
<thead>
<tr>
<th></th>
<th>yes</th>
<th>no</th>
</tr>
</thead>
<tbody>
<tr>
<td>female</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>male</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

13 females; 15 males

The condition male limits the sample space to 15 possible outcomes.

Of the 15 males, 7 did the dishes.

Therefore, $P(\text{wash the dishes} \mid \text{male}) = \frac{7}{15}$
Let’s Try One

Using the data in the table, find the probability that a sample of not recycled waste was plastic. \( P(\text{plastic} \mid \text{non-recycled}) \)

The given condition limits the sample space to non-recycled waste.

A favorable outcome is non-recycled plastic.

\[
P(\text{plastic} \mid \text{non-recycled}) = \frac{20.4}{48.9 + 10.1 + 9.1 + 20.4 + 67.8} = \frac{20.4}{156.3} \approx 0.13
\]

The probability that the non-recycled waste was plastic is about 13%. 

<table>
<thead>
<tr>
<th>Material</th>
<th>Recycled</th>
<th>Not Recycled</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paper</td>
<td>34.9</td>
<td>48.9</td>
</tr>
<tr>
<td>Metal</td>
<td>6.5</td>
<td>10.1</td>
</tr>
<tr>
<td>Glass</td>
<td>2.9</td>
<td>9.1</td>
</tr>
<tr>
<td>Plastic</td>
<td>1.1</td>
<td>20.4</td>
</tr>
<tr>
<td>Other</td>
<td>15.3</td>
<td>67.8</td>
</tr>
</tbody>
</table>
Conditional Probability Formula

- For any two events A and B from a sample space with $P(A)$ does not equal zero

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$
Using Tree Diagrams

Jim created the tree diagram after examining years of weather observations in his hometown. The diagram shows the probability of whether a day will begin clear or cloudy, and then the probability of rain on days that begin clear and cloudy.

a. Find the probability that a day will start out clear, and then will rain.

The path containing clear and rain represents days that start out clear and then will rain.

\[
P(\text{clear and rain}) = P(\text{rain} | \text{clear}) \cdot P(\text{clear})
\]

\[
= 0.04 \cdot 0.28
\]

\[
= 0.011
\]

The probability that a day will start out clear and then rain is about 1%.
b. Find the probability that it will not rain on any given day.

The paths containing clear and no rain and cloudy and no rain both represent a day when it will not rain. Find the probability for both paths and add them.

\[ P(\text{clear and no rain}) + P(\text{cloudy and no rain}) = P(\text{clear}) \cdot P(\text{no rain | clear}) + P(\text{cloudy}) \cdot P(\text{no rain | cloudy}) \]
\[ = 0.28(.96) + .72(.69) \]
\[ = 0.28 \cdot 0.96 + 0.72 \cdot 0.69 \]
\[ = 0.7656 \]

The probability that it will not rain on any given day is about 77%.
Let’s Try One

- A survey of Pleasanton Teenagers was given.
  - 60% of the responders have 1 sibling; 20% have 2 or more siblings
  - Of the responders with 0 siblings, 90% have their own room
  - Of the respondents with 1 sibling, 20% do not have their own room
  - Of the respondents with 2 siblings, 50% have their own room

Create a tree diagram and determine

A) \( P(\text{own room} \mid 0 \text{ siblings}) \)
B) \( P(\text{share room} \mid 1 \text{ sibling}) \)