Graph - a collection of vertices $V$ and edges $E$ (connection between a pair of vertices).

Modelling Fads
Today we’re going to model the spread of fads/opinions among a group. Our vertices are going to be people and there is an edge between them if they are friends. A vertex colored red represents a person who has adopted the trend, and a vertex colored purple represents one who has not yet adopted the trend.

Definitions:
- early adopters:
- trend-strength:
- payoff function:

Example: David has started drinking lattes. The strength of this trend is 0.6. Use the pay-off function to calculate whether his friends should hop on the trend or not. Based on today, will David have a higher pay-off if he drinks lattes or doesn’t?

If he drinks lattes tomorrow, he expects his net pay-off to be:

If he doesn’t drink lattes tomorrow, he expects his net pay-off to be:

Example: Here $t = .75$
Activity 1: Let \( d(R) \) be the number of red neighbors of a vertex \( v \), and let \( d(P) \) be the number of purple neighbors of \( v \). Note that the number of neighbors, \( n \), of \( v \) is \( n = d(R) + d(P) \). Write the net-payoff of \( v \) in terms of \( d(B) \) and \( d(R) \). Compare the net-payoff for choosing red and that for choosing purple, and figure out an easy test for when \( v \) chooses to be red.

Activity 2: Oh no! You have just been admitted in an infinite prison! Everyone is friends with their neighbors, because those are the only people that they can reach to talk to. When you enter, you decided to draw on your walls. No one else draws on the walls. This trend spreads with strength \( t = 0.5 \). What happens to your trend over time? Can you figure out what happens for \( t < 0.5 \) and for \( t > 0.5 \)? Here’s a picture of the prison:

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What happens if you and I are both admitted, we’re put in adjacent cells, and I share your penchant for drawing on the walls?

The contagion threshold:

Refining the model: You may be friends with Amy and Bob, but if you talk with Amy once a month and Bob every day, it is likely that they will not exert the same influence on you. How could we change our model to show this difference in influence? (Let’s say that we can assign values to people’s influence, where the most anyone can influence you has value 1, and if someone doesn’t influence you at all, influence has value 0.)

Activity 3: For any strength \( t \), find a weight \( w \) such that the trend in activity 2 will spread throughout the graph.