1 Activity I - The One with Cobweb Diagrams (25 min)

- Split into Groups of 4
- Every group gets a calculator and pens
- Guided walkthrough with first sheet. Introduce some vocabulary,
  - Iteration - An application of the map $f(x)$.
  - Iterate - The value of $x$ after an application or iteration of the map $f(x)$.
  - Fixed Points - Values of $x$ that are unchanged after a single iteration of $f(x)$.
  - Accumulating Points - Values of $x$ that appear to accumulate iterations
- Group work for next three sheets
  - Have everyone start somewhere different
  - Begin taping up Gallery
- Groups Share Results with Room. Guiding Questions:
  - What do the iterates look like?
  - Are there any fixed points?
  - Is the fixed point attractive?
- Discussion Time
  - What do the cobweb diagrams allow you to do?
  - What’s a good rule of thumb for fixed points?
  - How can you tell if a fixed point is attracting? repelling?
  - Other kinds of features? (periodic orbits?)

2 Activity II - The One with the Logistic Map (25 min)

- Finish taping up Gallery of unfinished cobwebs
- Focus on specific map: logistic map (sheet 4)
  - Models population growth in terms of Capacity
    - A fish tank can only fit so many fish before overcrowding becomes a problem.
    - Say you start with some percentage of the tank full of fish.
    - Let $r$ be the amount of feed you give the fish every week.
    - After one week, $f(x) = r(1 - x)x$ describes what percentage of the tank is full of fish.
– Big Question 1: Does it matter how many fish start in the tank?
– Big Question 2: Does it matter how much you feed the fish?
– Have pairs come up and grab a worksheet to cobweb
– Gallery Walkthrough
  * Do we have answers to the Big Questions?
  * Do we see periodic orbits?
  * How can we capture all of the information in the gallery into a hyper-diagram?
– Show Bifurcation Diagram Gif and Image. What do we observe in diagram?
  * The points are the accumulation points of the repeated iterations. They are what the cobwebs approach.
  * The points for a single $r$ value form a periodic orbit: because if you start at one of the points you periodically visit every other point in orbit.
  * There are repeated instances of branching, call these period doubling events.
  * A period doubling causes a single accumulation point to split into two, those doubling the period of these orbits.
  * With enough period doublings, the entire space is visited and we incur the onset of chaos.
  * There do exist some islands of stability where chaos recedes and we get periodic orbits again $r = 1 + \sqrt{8}$

3 Activity III - The One with Demos (10 min)

- Arnold’s Cat Map: https://www.jasondavies.com/catmap/ with image in Dropbox folder
  – Stretches image diagonally and folds it back around to fit a square.
  – Stretching and Folding is a common way to generate chaos.
  – After a few iterations, it becomes hard to identify what the image was in the beginning.
  – Onset of chaos does not mean loss of information or loss of structure, as a few more iterations and the image returns.
- Lorenz Attractor Example: Tracing Trajectories
  – Continuous example, where instead of say sampling the fish tank every week, we can track the fish population at all times.
  – This model is infamous and is used to explain the limitations of forecasting weather.
  – Video: https://www.youtube.com/watch?v=FD2GdjWUKuc @ x2 speed
    * The video begins with a line of nearby points and look at their evolution.
    * Early on, line is just stretched a bit as it goes around the attractor.
    * Later, the line is completely gone as points are spread out across attractor.
    * Later, the line comes back somewhat partially after some time.
  – Video: https://www.youtube.com/watch?v=FYE4JKAXSfY @ x2 speed. Same idea, but with rotation to see the 3D structure of attractor.
  – Video: https://www.youtube.com/watch?v=6lsP.iKms. See the time series observation of the Lorenz attractor