

## Homework for 651

Problems due May 3, 2007

1. Find a finite 2-dimensional complex that has no topological embedding in  $\mathbb{R}^3$ .
2. Problem 4, page 257, in Hatcher.
3. Problem 7, page 258, in Hatcher.
4. Problem 11, page 258 in Hatcher.
5. Show that every every piecewise linear embedding of a finite one-dimensional complex in the 5-sphere  $S^5$  is such that the complement is simply connected. (Regard  $S^5$  as the boundary of a 6-dimensional simplex, say.)
6. Show that the double suspension of a non-simply connected homology 3-sphere  $M^3$  is simply connected and has the homology and cohomology of the 5-sphere  $S^5$ . ( $M^3$  is a closed 3-dimensional manifold with the homology of the 3-sphere.)
7. In the 1970's Bob Edwards showed that the double suspension of a homology sphere  $\Sigma^2 M^3$  is homeomorphic to the 5-sphere. The non-simply connected homology 3-sphere  $M^3$  can be taken to be a simplicial complex. Show that Edward's homeomorphism to  $S^5$  is such that there is a circle that is a subcomplex of  $\Sigma^2 M^3$  but that its image  $C$  in  $S^5$  is wild; that is, the pair  $(S^5, C)$  is not homeomorphic to the standard  $(S^5, S^1)$ .