Homework for 651

Problems due April 17, 2007

1. Problem 19, page 157, in Hatcher.

2. Problem 32, page 158, in Hatcher.

3. Problem 36, page 158, in Hatcher.

4. One way to modify a presentation $< g_1, ..., g_m | r_1, ..., r_n >$ to another presentation for the same group is to replace r_i with $r_i r_j$, $r_i r_j^{-1}$, $r_j r_i$, or $r_j^{-1} r_i$ for some $j \neq i$. Show that the 2-complexes X_G associated to these different presentations are homotopy equivalent. [Hint: Attach the 2-cell e_i^2 last and deform its attaching map so as to change r_i to one of the new relations, then apply Proposition 0.18.]

5. Let $g : S^n \times I \to S^n \times I$ be continuous and $n > 0$. Let $X_k = \{(x, k) \mid x \in S^n\}$. For each fixed k, let $g(X_k) \subset X_k$. Further let $f_1 : S^1 \to S^1$ be the map $f_1(x) = 2x$. Define f_i inductively as $f_{i+1} = Sf_i$ where Sf_i is the suspension of the map f_i (see Hacther page 9.) Let $g((x, 0)) = f_n$. Suppose $g((x, 1)) = (x, 1)$, does such a map g exist? What if $g|_{X_1}$ is an odd map? What if $g|_{X_1}$ is an even map and n is odd? What if $g|_{X_1}$ is an even map and n is even? In each case, if a map g exists then give an example, if not prove that it does not exist.