1. (a) 
\[ \int \frac{1}{1 + \sin r} \, dr = \int \frac{1 - \sin^2 r}{1 - \sin^2 r} \, dr \]
\[ = \int \frac{1 - \sin r}{\cos^2 r} \, dr \]
\[ = \int \sec^2 r - \sec r \tan r \, dr \]
\[ = \tan r - \sec r + C. \]

(b) 
\[ \int_2^5 (t - 2)^{-2/3} \, dt = \lim_{a \to 2^+} \int_a^5 (t - 2)^{-2/3} \, dt \]
\[ = \lim_{a \to 2^+} \int_{a-2}^3 u^{-2/3} \, du \quad \text{(sub } u = t - 2) \]
\[ = \lim_{a \to 2^+} \left[ 3u^{1/3} \right]_{a-2}^3 \]
\[ = 3\sqrt[3]{3}. \]

2. (a) \[ \frac{x^4 + 2x^2 - x + 1}{(x^2 + 1)^2 x} = \frac{Ax + B}{x^2 + 1} + \frac{Cx + D}{(x^2 + 1)^2} + \frac{E}{x}. \]
(b) 
\[ \int \frac{x}{(x^2 + 1)^2} \, dx = \frac{1}{2} \int u^{-2} \, du \quad \text{(sub } u = x^2 + 1) \]
\[ = -\frac{1}{2(x^2 + 1)} + C. \]

To find \[ \int \frac{1}{(x^2 + 1)^2} \, dx \]: let \( x = \tan \theta \). Then \( x^2 + 1 = \sec^2 \theta \), \( dx = \sec^2 \theta \, d\theta \). The integral becomes
\[ \int \frac{d\theta}{\sec^2 \theta} = \int \cos^2 \theta \, d\theta \]
\[ = \int \frac{1}{2} + \frac{1}{2} \cos(2\theta) \, d\theta \]
\[ = \frac{1}{2} \theta + \frac{1}{4} \sin(2\theta) + C \]
\[ = \frac{1}{2} \tan^{-1} x + \frac{x}{2(1 + x^2)} + C. \]

(Using \( \sin(2\theta) = 2 \sin \theta \cos \theta \) and a right triangle with sides \( x, 1, \sqrt{1 + x^2} \).) Putting these together,
\[ \int \frac{x + 1}{(x^2 + 1)^2} \, dx = \frac{x - 1}{2(x^2 + 1)} + \frac{1}{2} \tan^{-1} x + C. \]
3. Let \( R \) be the region under the graph of \( y = xe^x \) (and above the \( x \)-axis) for \( x \in [0, 1] \). Find the volume of the solid obtained by revolving \( R \) about the \( x \)-axis.

The volume is given by
\[
\pi \int_0^1 x^2 e^{2x} \, dx
\]
\[
\int_0^1 x e^{2x} \, dx = \frac{e^2}{2}
\]
\[
\pi \left[ \frac{1}{2} x e^{2x} \right]_0^1 = \pi \int_0^1 x e^{2x} \, dx
\]
\[
= \pi \left( \frac{e^2}{2} - \pi \int_0^1 x e^{2x} \, dx \right)
\]
\[
= \frac{\pi e^2}{2} - \pi \left( \frac{e^2}{2} \right)_0^1 = \pi \left( e^2 - \frac{1}{4} \right).
\]

4. (a)
\[
\lim_{n \to \infty} a_n = \lim_{n \to \infty} \ln\left(\frac{n}{2n}\right)
\]
\[
= \lim_{n \to \infty} \frac{\ln n}{2n}
\]
by L'Hôpital's Rule
\[
= \lim_{n \to \infty} \frac{1}{2n} = 0.
\]

(b) Since \( 3 \leq 4 + \cos(3n) \leq 5 \), it follows that
\[3a_n \leq b_n \leq 5a_n,\]
and so \( b_n \) converges with limit zero by the Squeeze Theorem.

(c) Let \( c_n \) be the sequence defined by
\[
c_1 = a_1
\]
\[
c_n = a_n - a_{n-1} \quad \text{for } n > 1.
\]
Does the series \( \sum_{n=1}^{\infty} c_n \) converge?
Consider the partial sums:
\[
S_1 = c_1 = a_1
\]
\[
S_2 = c_1 + c_2 = a_1 + (a_2 - a_1) = a_2
\]
\[
\vdots
\]
\[
S_n = a_1 + (a_2 - a_1) + (a_3 - a_2) + \cdots + (a_n - a_{n-1}) = a_n.
\]
So the series converges (with sum $\sum_{i=1}^{\infty} c_n = 0$).

5. 

(a) $\sum_{n=1}^{\infty} \frac{3^{n+1}}{(-2)^{3n+3}} = \sum_{n=1}^{\infty} \frac{3^{n+1}}{((-2)^3)^{n+1}} = \sum_{n=1}^{\infty} \left( -\frac{3}{8} \right)^{n+1}$. 
This is a geometric series with sum 
$$a \frac{1}{1-r} = \frac{8}{11}.$$ 

(b) $\sum_{n=1}^{\infty} \ln \sqrt{n+1} = \sum_{n=1}^{\infty} \ln \left( \frac{n}{n+1} \right)^{1/2} = \frac{1}{2} \sum_{n=1}^{\infty} (\ln n - \ln(n+1))$. 
This is a telescoping sum; the partial sums are given by 
$$S_n = -\frac{1}{2} \ln(n+1),$$ 
so the series diverges.

6. Are the following series convergent or divergent?

(a) $\sum_{n=15}^{\infty} \frac{\ln n}{n^{5/2}}$. This is convergent; use limit comparison test with $a_n = \frac{\ln n}{n^{7/2}}$, and $b_n = \frac{1}{n^{5/2}}$. 
The series $\sum b_n$ is a $p$-series with $p > 1$ so it converges. The limit $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\ln n}{n} = 0$ 
(which we showed in class using L'Hôpital).

(b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + n^5}$ 
Convergent; use comparison test (direct or limit), compare to $\sum \frac{1}{n^5}$.

(c) $\sum_{n=-3}^{\infty} e^{-n^2}$

This is convergent. First use the direct comparison test, comparing $\sum_{n=1}^{\infty} e^{-n^2}$ to $\sum_{n=1}^{\infty} e^{-n}$ 
(you can forget about the first 4 terms which have a finite sum). Then use the integral test to show $\sum_{n=1}^{\infty} e^{-n}$ is convergent (noting that $f(x) = e^{-x}$ is positive, continuous and decreasing).